Christ Church Grammar School

2016
UNIT TEST 1

MATHEMATICS METHODS Year 12

Section One: Calculator-free

Student name Solution

Teacher name _____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(4 marks)**

Differentiate with respect to x. Do not simplify your answers.

(a) x^2e^{-2x} (1 mark)

$\frac{d}{dx} x^2e^{-2x} = 2xe^{-2x} + x^2 \cdot e^{-2x} \cdot (-2)$ ✓

(b) $e^{\tan 2x}$ (1 mark)

$\frac{d}{dx} e^{\tan 2x} = e^{\tan 2x} \cdot \sec^2 2x \cdot 2$

or $= e^{\tan 2x} \cdot \frac{1}{\cos^2 2x} \cdot 2$ ✓

(c) $\frac{\sqrt{x}}{\cos(2x-1)}$ (2 marks)

$\frac{d}{dx} \frac{\sqrt{x}}{\cos(2x-1)} = \frac{\frac{1}{2\sqrt{x}} \cos(2x-1) + \sqrt{x} \sin(2x-1) \cdot 2}{\cos^2(2x-1)}$ ✓ ✓

Question 2 **(3 marks)**

Find $\frac{dy}{dx}$ given that $x = e^{\sin\theta}$ and $y = e^{\cos\theta}$.

$$\frac{dx}{d\theta} = e^{\sin\theta} \cdot \cos\theta \quad \checkmark$$

$$\frac{dy}{d\theta} = e^{\cos\theta} \cdot (-\sin\theta) \quad \checkmark$$

$$\frac{dy}{dx} = \frac{e^{\cos\theta}(-\sin\theta)}{e^{\sin\theta} \cdot \cos\theta} \quad \checkmark$$

$$= -e^{\cos\theta - \sin\theta} \tan\theta .$$

Question 3 **(4 marks)**

Find the minimum and maximum values of $f(x) = \frac{x^3}{3} - x^2 + 4$ over the interval $-3 \leq x \leq 3$.

$f'(x) = x^2 - 2x$

$f'(x) = 0 \Rightarrow x^2 - 2x = 0$ ✓

$x(x-2) = 0$

$x = 0$ or 2

$f(0) = 4$
$f(2) = \frac{8}{3}$ } ✓

$f(-3) = -14$
$f(3) = 4$ } ✓

$\therefore$ max $= 4$
min $= -14$ } ✓

See next page

Question 4 **(4 marks)**

For the function $f(x) = (x - 200)^6 + 300$,

(a) find the value of a for which $f''(a) = 0$ (1 mark)

$f'(x) = 6(x-200)^5$

$f''(x) = 30(x-200)^4$

$\therefore a = 200$ ✓

(b) determine the concavity of $y = f(x)$ when $x < a$ and when $x > a$ (2 marks)

when $x < 200$, $f''(x) > 0$

$\Rightarrow$ curve is concave up ✓

when $x > 200$, $f''(x) > 0$

$\Rightarrow$ curve is concave up ✓

(c) hence determine if $x = a$ is a point of inflection or not, giving a reason for your answer.

(1 mark)

$x = 200$ is not a P.O.I. ✓

since there is no change in concavity

from $x < 200$ to $x > 200$.

End of questions

Christ Church Grammar School

2016
UNIT TEST 1

MATHEMATICS METHODS Year 12

Section Two: Calculator-assumed

Student name Solution

Teacher name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

Important note to candidates

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Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

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See next page

Question 5 **(8 marks)**

(a) Use the method of small changes to find the approximate change in the radius of a spherical balloon corresponding to a change in its volume from $500\ cm^3$ to $485\ cm^3$. (4 marks)

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$ ✓

$\frac{4}{3}\pi r^3 = 500$

$r \approx 4.924\ cm$ ✓

$\delta V \approx 4\pi r^2 \delta r$

$-15 = 4\pi(4.924)^2 \delta r$ ✓

$\delta r \approx -0.049\ cm$ ✓

(b) The displacement of a body at time t seconds is given by $x = 4t + \frac{1}{1+t}$ metres. Find an expression for the velocity of the body at time t seconds and then show that the body is never stationary. (4 marks)

$v(t) = 4 - \frac{1}{(1+t)^2}\ ms^{-1}$ ✓

At stationary point, $v(t) = 0$ ✓

$t = -\frac{3}{2}$ or $-\frac{1}{2}$ ✓

But $t \geq 0$

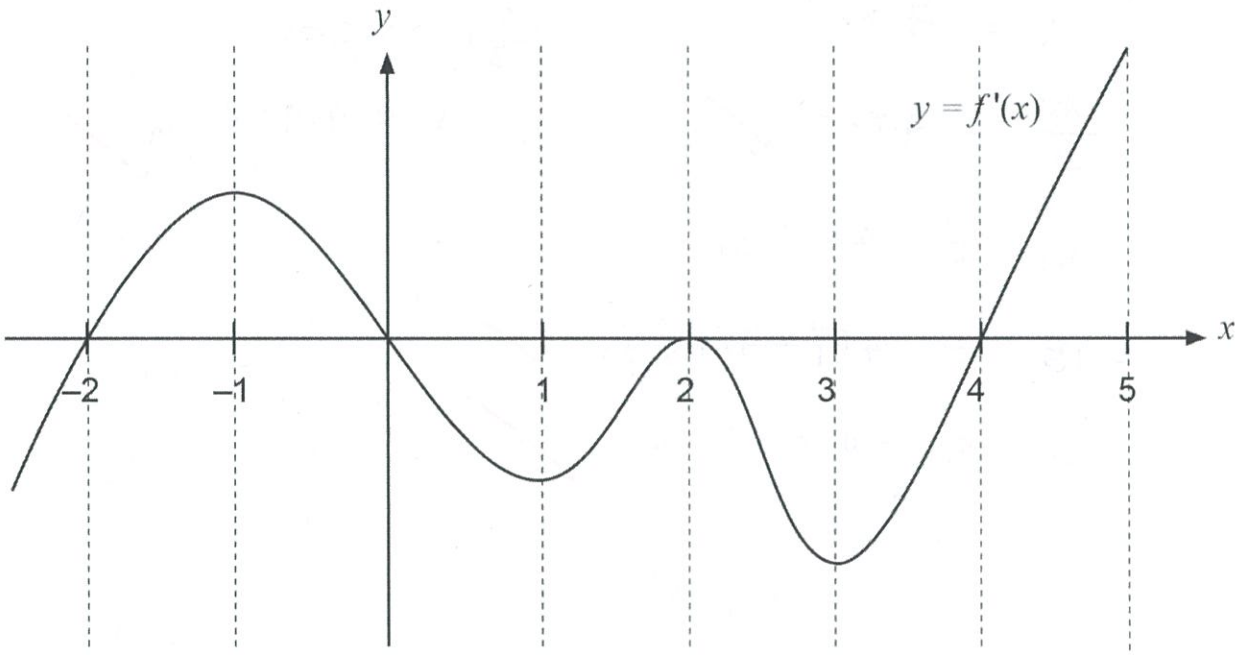
$\therefore v(t) \neq 0$ for all $t \geq 0$ ✓

$\Rightarrow$ body is never stationary

See next page

Question 6 **(9 marks)**

The diagram below shows the graph of $y = f'(x)$ of a function $y = f(x)$.



(a) For what values of x does $y = f(x)$ have a local maximum or minimum? (2 marks)

local max at $x = 0$ ✓

local min at $x = -2$ or 4 ✓

(if $x = 2$ is included, that point is marked wrong.)

(b) For what values of x does $y = f(x)$ have inflection points? (2 marks)

P.O.I. at $x = -1, 1, 2, 3$ ✓ ✓

(one mark for every 2 correct)

(c) Does $y = f(x)$ have a horizontal point of inflection? Explain (2 marks)

Yes. ✓

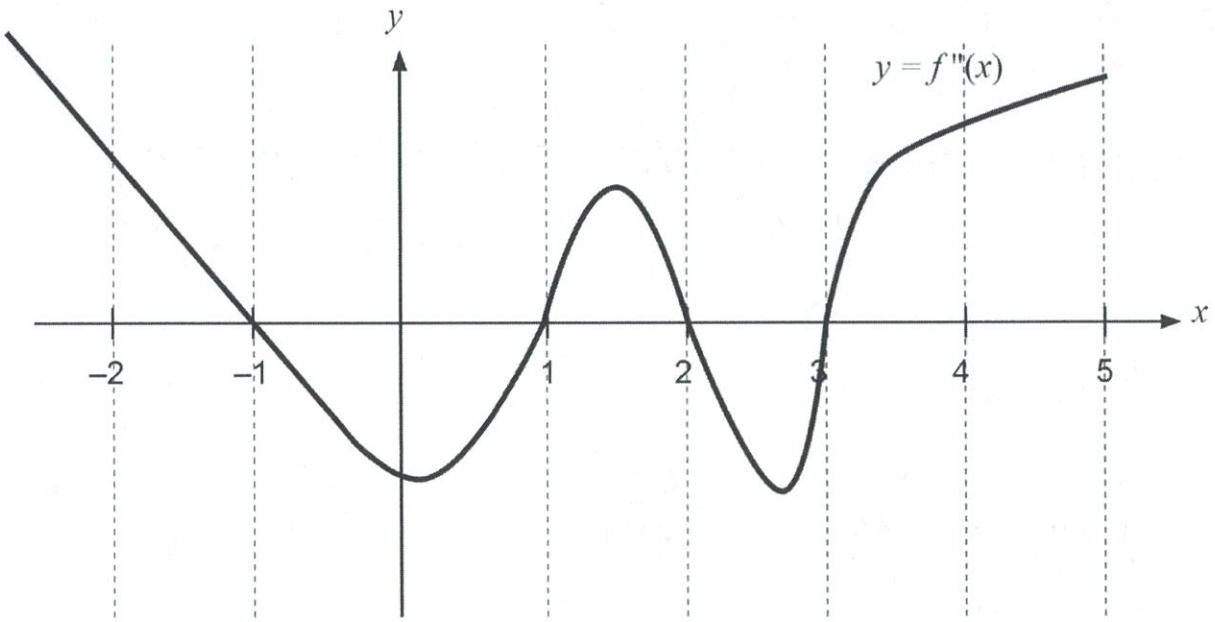
$x = 2$ is a horizontal P.O.I.

since $f'(x) = 0$ and $f''(x) = 0$ ✓

See next page

Question 6 continued

(d) On the axis below, sketch the graph of $y = f''(x)$. (3 marks)



✓ show all four x-intercepts

✓ show 3 turning points in the correct intervals

✓ show a negative y-intercept

Question 7 **(7 marks)**

KSL Productions sells a product at a unit price of \$30. The cost of producing x items is given by $C(x) = \frac{80x}{x+1} + 0.04x^2 + 500$.

(a) Find an expression for the profit $P(x)$ corresponding to the manufacture and sale of x items. (1 mark)

$P(x) = 30x - \left(\frac{80x}{x+1} + 0.04x^2 + 500\right)$ ✓

(b) Find an expression $P'(x)$. (1 mark)

$P'(x) = 30 - \frac{80}{(x+1)^2} - 0.08x$ ✓

(c) Find $P'(100)$. Interpret this value. (2 marks)

$P'(100) \approx \$22$ ✓

$\Rightarrow$ Profit on the 101st item is \$22 ✓

(d) Find the average profit per item associated with the manufacture and sale of 100 items. (1 mark)

Average profit $= \frac{P(100)}{100}$

$= \$20.21$ ✓

Question 7 continued

(e) Find how many items were manufactured and sold if the profit associated with the sale of the next item is approximately \$10, given that more than 100 items were manufactured and sold. (2 marks)

$P'(x) = 10$ ✓

$x = 1$ or 250

$\therefore$ 250 ✓ items were manufactured & sold.

See next page

Question 8 **(6 marks)**

Consider two circles, the first having a radius r_1 and the other radius r_2 , with the sum of the two radii being constant, that is, $r_1 + r_2 = c$.

(a) Find an expression for the sum of the areas of the two circles in terms of r_1 and c. (2 marks)

$$A = \pi r_1^2 + \pi r_2^2 \quad \checkmark$$

$$= \pi r_1^2 + \pi (c - r_1)^2 \quad \checkmark$$

(b) Use calculus to prove that if the sum of the radii of two circles is constant, then the sum of the areas of the two circles is at a minimum when the circles have equal radii. (4 marks)

$$\frac{dA}{dr_1} = 2\pi r_1 + 2\pi(c - r_1)(-1) \quad \checkmark$$

$$= 4\pi r_1 - 2\pi c$$

$$\frac{dA}{dr_1} = 0$$

$$\Rightarrow \quad 4\pi r_1 = 2\pi c$$

$$r_1 = \tfrac{1}{2}c \quad \checkmark$$

$$r_2 = \tfrac{1}{2}c$$

$$\frac{d^2A}{dr_1^2} = 4\pi > 0 \quad \checkmark$$

$$\Rightarrow \quad A \text{ is a minimum}$$

$$\text{when } r_1 = r_2 = \tfrac{1}{2}c. \quad \checkmark$$

End of questions